# Silicon-Oxygen Bond Lengths, Bridging Angles Si-O-Si and Synthetic Low Tridymite 

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#### Abstract

The crystal structure of synthetic low tridymite has been re-refined to an $R=0.076$ for $4117 F$ (obs) with data collected by Kato \& Nukui [Acta Cryst. (1976), B32, 2486-2491]. The structure is monoclinic, space group $C c$, with $a=18.494$ (8), $b=4.991(2), c=23.758$ (8) $\AA, \beta=105.79(2)^{\circ}, Z=12$. The mean $\mathrm{Si}-\mathrm{O}$ distance measures 1.597 (10) $\AA$ and the mean $\mathrm{Si-O}-\mathrm{Si}$ angle is $150.0(7.4)^{\circ}$. The structure of this synthetic Cc low tridymite and that of the recently refined meteoritic low tridymite are identical within the margin of experimental error. A regression analysis of data from tridymite, quartz, cristobalite, coesite, zunyite, hemimorphite, benitoite, beryl, emerald, $\mathrm{Sc}_{2} \mathrm{Si}_{2} \mathrm{O}_{7}, \mathrm{Yb}_{2} \mathrm{Si}_{2} \mathrm{O}_{7}$ and $\mathrm{Er}_{2} \mathrm{Si}_{2} \mathrm{O}_{7}$ shows that the correlation coefficient between the $\mathrm{Si}-\mathrm{O}$ bond length and the negative secant of the angle $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ is $0 \cdot 19$; consequently only $4 \%$ of the variation in $\mathrm{Si}-\mathrm{O}$ can be explained by the dependence on $-\mathrm{sec}(\mathrm{Si}-\mathrm{O}-\mathrm{Si})$. Therefore the null hypothesis that the $\mathrm{Si}-\mathrm{O}$ bond lengths do not depend on $-\sec (\mathrm{Si}-\mathrm{O}-\mathrm{Si})$ can be accepted at the $2 \cdot 5 \%$ risk level. For a given $\mathrm{Si}-\mathrm{O}$ bond length the lower limit of possible $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ angles is determined by the shortest possible non-bonding $\mathrm{Si}-\mathrm{Si}$ contact. Therefore this lower limit is a function of the $\mathrm{Si}-\mathrm{O}$ bond length and not vice versa as required by $\pi$-bonding theory.


## Introduction

The crystal structure of meteoritic low tridymite has recently been solved by Dollase \& Baur (1976) in space group $C c$, with $a=18.524$ (4), $b=5.0032$ (15), $c=$ 23.810 (8) $\AA, \beta=105.82(2)^{\circ}, Z=12$. The refinement of 1260 Mo $K \alpha$ reflections gave an $R=0.069$. The 48 individual, crystallographically independent $\mathrm{Si}-\mathrm{O}$ distances range from 1.55 to $1.65 \AA$, with a mean value of $1.601 \AA$, and a standard deviation from this mean of $0.017 \AA$. Kato \& Nukui (1976) determined the structure of the corresponding synthetic tridymite and refined it for 4117 Mo $K \alpha$ reflexions to $R=0.085$ [ $C c$, $a=18.494$ (8), $b=4.991$ (2), $c=25.832$ ( 8 ) $\AA, \beta=$ $117.75(2)^{\circ}, \quad Z=12$ ]. The 48 individual $\mathrm{Si}-\mathrm{O}$ distances range from 1.552 to $1.634 \AA$, with a mean value of 1.595 (20) $\AA$. The data used by Dollase \& Baur were collected in a range up to $\sin \theta / \lambda=0.7 \AA^{-1}$. The Kato \& Nukui data range up to a $\sin \theta / \lambda=1 \cdot 0$ $\AA^{-1}$, and their refinement should be more reliable. However, the standard deviation from Kato \& Nukui's mean $\mathrm{Si}-\mathrm{O}$ value is larger than for the mean value obtained by Dollase \& Baur. Because of computermemory limitations Kato \& Nukui (1976) could not refine all parameters in the same cycle. This might have hindered proper convergence.

## Re-refinement of synthetic tridymite

The two different unit cells used for meteoritic and synthetic tridymite are related by the transformation matrix $(1,0,0 / 0,-1,0 /-1,0,-1)$. Since the choice of origin in $C c$ is arbitrary for $x$ and $z$ the relationships for the positional parameters are $x_{\mathrm{DB}}=x_{\mathrm{KN}}-z_{\mathrm{KN}}-$ $0.2606, y_{\mathrm{DB}}=0.75-y_{\mathrm{KN}}$ and $z_{\mathrm{DB}}=-z_{\mathrm{KN}}$ (where DB and KN refer to the Dollase \& Baur and Kato \& Nukui settings respectively). The re-refinement of Kato \& Nukui's data was performed in the setting of Dollase \& Baur. In this setting the dimensions of the unit cell of synthetic tridymite are: $a=18.494$ (8), $b=4.991$ (2), $c=23.758$ (8) $\AA, \beta=105.79$ (2) ${ }^{\circ}$. The scattering factors of $\mathrm{Si}^{2+}$ and $\mathrm{O}^{-}$were taken from International Tables for $X$-ray Crystallography (1974). A simultaneous, full-matrix, unit-weight refinement of all 323 adjustable parameters yielded in three cycles an $R=$ 0.076 for $4117 F_{\text {obs }}$.

## Discussion of the structure

The new positional parameters (Table 1) lead to $\mathrm{Si}-\mathrm{O}$ distances (Table 2) ranging from 1.576 to $1.622 \AA$, with a mean of 1.597 (10) $\AA$. The range of $0.046 \AA$ is appreciably smaller than for either the Dollase \& Baur $(0 \cdot 10 \AA)$ or the Kato $\&$ Nukui $(0.082 \AA)$ refinements. It is only slightly larger than the range of $0.033 \AA$

Table 1. Synthetic low tridymite: positional and thermal parameters $\left(\times 10^{4}\right)$

The temperature factor is $\exp \left[-\left(\beta_{11} h^{2}+\beta_{22} k^{2}+\beta_{33} l^{2}+2 \beta_{12} h k+2 \beta_{13} h l+2 \beta_{23} k l\right)\right]$. KN\# indicates the numbers of the atoms used by Kato \& Nukui (1976). The $x$ and $z$ parameters of $\mathrm{Si}(11)$ were not varied, in order to define the origin (*).

|  | KN\# | x | y | $z$ | ${ }^{\beta} 11$ | $8_{22}$ | $\beta_{33}$ | ${ }^{\beta} 12$ | ${ }^{\beta} 13$ | $\mathrm{B}_{23}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Si (1) | 6 | 5477.(2) | 5433 (7) | 5653(2) | 3 (1) | $87(7)$ | 4 (.3) | 0 (2) | 1 (.3) | $2(2)$ |
| Si (2) | 4 | 7003 (2) | -9458(7) | 7381 (2) | 7 (1) | 73 (8) | 4 (.3) | 3 (2) | $2(.3)$ | 7 (2) |
| Si (3) | 5 | 4171 (2) | 5508 (7) | 6236 (2) | 6 (1) | 82 (7) | 4 (.3) | -1(2) | 1 (.3) | -8(2) |
| Si (4) | 3 | 5724 (2) | 0464 (7) | 7977 (2) | 8 (1) | 74 (7) | 3 (.3) | 5 (2) | 1 (.3) | -4(2) |
| Si (5) | 10 | 9219 (2) | 5503(7) | 6987 (2) | 6 (1) | 79 (8) | 3 (.3) | -4 (2) | $1(.3)$ | -1(2) |
| Si (6) | 12 | 7622 (2) | 9414(7) | 5393 (2) | 6 (1) | 77 (8) | 4 (.3) | -1(2) | $10(.3)$ | -5(2) |
| Si (7) | 9 | 7997 (2) | 4474 (7) | 7670 (2) | 7 (1) | 85 (3) | 4 (.3) | 0 (2) | $3(.3)$ | ? (2) |
| Si (8) | 11 | 6428 (2) | 0393 (7) | 6042 (2) | $5(1)$ | 76 (7) | 3 (.3) | 6 (2) | 1 (.3) | -1(2) |
| Si (9) | 2 | 8493(2) | 4614 (7) | 9030 (2) | $5(1)$ | 94 (8) | 3(.3) | -2(2) | $0(.3)$ | 1 (2) |
| Si (10) | 7 | 9454 (2) | 9637 (7) | 9375 (2) | 6 (1) | 72 (8) | 3 (.3) | -5(2) | 0 (.3) | $0(2)$ |
| Si (11) | 1 | 7216* | 5607 (7) | 9618* | 5 (1) | 54 (8) | 4 (.3) | 2 (2) | $0(.3)$ | $2(2)$ |
| Si (12) | 8 | 5663 (2) | 5457 (7) | 8716 (2) | 6(1) | 88 (8) | 3 (.3) | -1(2) | 2 (.3) | -1(2) |
| O(1) | 5 | 5723 (6) | 3471(18) | 8204 (4) | 20 (3) | 137 (29) | 7 (1) | 3 (7) | 5 (1) | -4(5) |
| O(2) | 4 | 5665 (5) | 3457(15) | 8494 (4) | 18 (2) | 41 (20) | 9 (1) | 5 (5) | 7 (1) | 5 (4) |
| O(3) | 1 | 7297 (5) | 1394 (16) | 4864 (3) | 24 (3) | 60 (23) | 5 (1) | 1 (6) | $0(1)$ | 3(4) |
| O(4) | 2 | 7488 (5) | 6415 (16) | 5160 (4) | 13 (2) | 68 (23) | $9(1)$ | 3 (5) | -2(1) | $0(4)$ |
| O(5) | 8 | 4133 (6) | 3504 (16) | 6752 (4) | 25 (3) | 52 (22) | $9(1)$ | 1 (7) | $8(2)$ | -4(4) |
| O(6) | 7 | 4096(5) | $8505(16)$ | 6444 (3) | 16 (2) | $83(23)$ | 6 (1) | -7(1) | 0 (1) | -12(4) |
| $0(7)$ | 14 | 8748 (4) | 1625(16) | 9225 (4) | $6(1)$ | 108(25) | 9 (1) | 10(5) | $2(1)$ | $10(4)$ |
| O(8) | 13 | 9130(5) | 6633 (16) | 9375 (3) | 14 (2) | 76 (23) | 7 (1) | -15(5) | $0(1)$ | -1 (4) |
| O(9) | 17 | 7714 (5) | 1484 (18) | 7566 (4) | 12 (2) | $98(26)$ | 14 (2) | -8(6) | $2(2)$ | -4(6) |
| $\mathrm{O}(10)$ | 16 | 7302(5) | 6442(16) | 7437 (4) | 11 (2) | 62 (22) | 12 (2) | -2(5) | -1(1) | -10(5) |
| $0(11)$ | 20 | $6177(4)$ | 3391 (14) | 5856 (3) | 8(2) | $48(20)$ | 6 (1) | $8(4)$ | -1(1) | $3(4)$ |
| O(12) | 19 | 5789 (5) | 8410(16) | 5670 (4) | 14 (2) | 77 (24) | 10 (1) | -5 (6) | 1 (1) | 8(5) |
| O(13) | 12 | 4958 (4) | $5109(18)$ | 6083 (4) | $6(1)$ | $280(46)$ | 8 (1) | 4 (6) | 4 (1) | - 2 (5) |
| $\bigcirc(14)$ | 9 | 3506 (4) | 4920(17) | 5653 (4) | 5 (1) | 251 (44) | 7 (1) | -10(5) | 1 (1) | -20(5) |
| O(15) | 18 | 6524 (5) | $0009(21)$ | 6721 (4) | $10(2)$ | 300 (59) | 6 (1) | 21 (7) | 1(1) | 16 (5) |
| $0(16)$ | 11 | 6490 (5) | 9945 (21) | 7809 (4) | 11 (2) | 348 (51) | 5 (1) | 3 (7) | $4(1)$ | -8(5) |
| O(17) | 23 | 8607 (6) | 4970 (22) | 7324 (5) | $22(3)$ | $311(50)$ | 12 (2) | -28 (8) | 14(2) | -11(0) |
| O(18) | 15 | 8377 (6) | $5008(19)$ | 8341 (4) | 20 (3) | 284 (57) | $3(1)$ | 8(8) | $2(1)$ | $0(5)$ |
| $0(19)$ | 6 | 5040 (5) | 0017 (20) | 7408 (4) | 14 (2) | $250(36)$ | $8(1)$ | $6(7)$ | -2(1) | -9(5) |
| 0 (20) | 3 | 6355(4) | 4979(18) | 9279 (4) | 4 (1) | 240(45) | 7 (1) | -4(5) | -1(1) | 11 (5) |
| $0(21)$ | 22 | 9900 (5) | 9898 (21) | 8887 (4) | $8(2)$ | 333 (52) | 5 (1) | $-15(6)$ | $1(1)$ | f(5) |
| 0 (22) | 21 | 5003(5) | 4741(16) | 4999 (4) | 8 (2) | 168(24) | 4 (1) | $3(6)$ | $2(1)$ | -1(5) |
| O(23) | 24 | 7219 (5) | 9832(17) | 5902 (4) | $9(2)$ | 199(37) | 7 (1) | $9(5)$ | $2(1)$ | -6(4) |
| O(24) | 10 | 7712 (5) | 5206 (20) | 9163 (4) | $9(2)$ | 256(40) | $9(1)$ | $8(6)$ | $6(1)$ | 14 (5) |

observed for $\mathrm{Si}-\mathrm{O}$ distances in well refined $\mathrm{SiO}_{4}$ tetrahedra in which all O atoms are bridging between two Si atoms each (see entries with $\mathrm{NC}=4$ in Table 3). The mean change in $\mathrm{Si}-\mathrm{O}$ bond lengths achieved by the rerefinement is $0.012 \AA$, which is more than the estimated standard deviation for an individual $\mathrm{Si}-\mathrm{O}$ bond of $0.008 \AA$ reported by Kato \& Nukui (1976). Since the standard deviation from the mean of these bond lengths is now half as large as it was before the re-refinement, it is believed that the new results are physically meaningful. Kato \& Nukui (1976) noted the large thermal amplitudes of their atoms $\mathrm{O}(11), \mathrm{O}(12)$ and $O$ (18) and suggested that these might be due to positional disorder. The re-refinement of their data does not reveal anything particular about the thermal parameters of $\mathrm{O}(13), \mathrm{O}(15)$ and $\mathrm{O}(16)$ (using the numbering system of Table 1). It is remarkable that in the Kato \& Nukui refinement the ratio between the largest and the smallest $\beta_{22}$ parameter of the O atoms is 765 , whereas after re-refinement this ratio is reduced to

8 or by two orders of magnitude. Therefore it is not necessary to assume disorder for these atoms. An inspection of the $\beta_{22}$ parameters of the O atoms reveals that $\mathrm{O}(1)$ to $\mathrm{O}(12)$ have a small thermal amplitude parallel to the $b$ axis, whereas the remaining O atoms, $\mathrm{O}(13)$ to $\mathrm{O}(24)$, have a high amplitude in this direction. The latter group correspond to the bridging O atoms in the chains of $\mathrm{SiO}_{4}$ tetrahedra running parallel to the pseudohexagonal axis, $|201|$ |see Fig. 1 in Dollase \& Baur (1976)|.

With three exceptions the $\mathrm{Si}-\mathrm{O}$ distances and the $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ angles in meteoritic low tridymite (Dollase \& Baur, 1976) and in the re-refined synthetic low tridymite are within two pooled e.s.d.'s of each other. The exceptions are: $\mathrm{Si}(2)-\mathrm{O}(15)$ and the angles around $\mathrm{O}(15)$ and $\mathrm{O}(18)$. This close similarity between the results of the refinements indicates that the structures indeed are identical, even though their unit-cell constants differ slightly: up to six times the estimated standard deviations.

Table 2. Synthetic low tridymite, $\mathrm{Si}-\mathrm{O}$ bond lengths, $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ angles and $\mathrm{Si}-\mathrm{Si}$ distances
(1) Si O distances $(\AA)$ (mean e.s.d. $0.008 \AA$ )

| $\mathrm{Si}(1)-\mathrm{O}(13)$ | 1.589 | $\mathrm{Si}(5)-\mathrm{O}(17)$ | 1.576 | $\mathrm{Si}(9)-\mathrm{O}(24)$ | 1.588 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}(12)$ | 1.590 | O (5) | 1.591 | $\mathrm{O}(7)$ | 1.595 |
| $\mathrm{O}(22)$ | 1.600 | $\mathrm{O}(19)$ | 1.594 | $\mathrm{O}(8)$ | 1.596 |
| $\mathrm{O}(11)$ | 1.615 | O (6) | 1.598 | $\mathrm{O}(18)$ | 1.604 |
| mean | 1.599 | mean | 1.590 | mean | 1.596 |
| $\mathrm{Si}(2)-\mathrm{O}(16)$ | 1.586 | $\mathrm{Si}(6)-\mathrm{O}(3)$ | 1.584 | $\mathrm{Si}(10)-\mathrm{O}(22)$ | 1.584 |
| $\mathrm{O}(10)$ | 1.597 | $\mathrm{O}(4)$ | 1.592 | $\mathrm{O}(21)$ | 1.600 |
| $\mathrm{O}(15)$ | 1.602 | $\mathrm{O}(23)$ | 1.599 | $\mathrm{O}(7)$ | 1.601 |
| $\mathrm{O}(9)$ | 1.622 | $\mathrm{O}(14)$ | 1.603 | $\mathrm{O}(8)$ | 1.615 |
| mean | 1.602 | mean | 1.595 | mean | 1.600 |
| $\mathrm{Si}(3)-\mathrm{O}(6)$ | 1.593 | $\mathrm{Si}(7)-\mathrm{O}(9)$ | 1.578 | $\mathrm{Si}(11)-\mathrm{O}(3)$ | 1.599 |
| O (5) | 1.599 | O (18) | 1.581 | $\mathrm{O}(4)$ | 1.606 |
| $\mathrm{O}(13)$ | 1.606 | O(17) | 1.586 | $\mathrm{O}(20)$ | 1.608 |
| $\mathrm{O}(14)$ | 1.608 | $\mathrm{O}(10)$ | 1.592 | $\mathrm{O}(24)$ | 1.609 |
| mean | 1.602 | mean | 1.584 | mean | 1.606 |
| $\mathrm{Si}(4)-\mathrm{O}(16)$ | 1.594 | $\mathrm{Si}(8)-\mathrm{O}(15)$ | 1.585 | $\mathrm{Si}(12)-\mathrm{O}(2)$ | 1.588 |
| $\mathrm{O}(1)$ | 1.594 | O(11) | 1.593 | $\mathrm{O}(21)$ | 1.596 |
| O (19) | 1.596 | $\mathrm{O}(23)$ | $1 \cdot 606$ | $\mathrm{O}(20)$ | 1.597 |
| O (2) | 1.611 | $\mathrm{O}(12)$ | 1.609 | $\mathrm{O}(1)$ | 1.598 |
| mean | 1.599 | mean | 1.598 | mean | 1.595 |

(2) $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ angles $\left(^{\circ}\right.$ ) (mean e.s.d. $0 \cdot 6^{\circ}$ ) and corresponding $\mathrm{Si}-\mathrm{Si}$ distances $(\AA)$ (mean e.s.d. $0 \cdot 004 \AA$ )

| $\mathrm{Si}(4)-\mathrm{O}(1)-\mathrm{Si}(12)$ | 148.0 | 3.069 |
| :--- | :--- | :--- |
| $\mathrm{Si}(12)-\mathrm{O}(2)-\mathrm{Si}(4)$ | 147.8 | 3.074 |
| $\mathrm{Si}(6)-\mathrm{O}(3)-\mathrm{Si}(11)$ | 148.5 | 3.063 |
| $\mathrm{Si}(6)-\mathrm{O}(4)-\mathrm{Si}(11)$ | 148.9 | 3.080 |
| $\mathrm{Si}(5)-\mathrm{O}(5)-\mathrm{Si}(3)$ | 146.9 | 3.058 |
| $\mathrm{Si}(3)-\mathrm{O}(6)-\mathrm{Si}(5)$ | 146.3 | 3.054 |
| $\mathrm{Si}(9)-\mathrm{O}(7)-\mathrm{Si}(10)$ | 143.4 | 3.034 |
| $\mathrm{Si}(9)-\mathrm{O}(8)-\mathrm{Si}(10)$ | 143.9 | 3.053 |
| $\mathrm{Si}(7)-\mathrm{O}(9)-\mathrm{Si}(2)$ | 147.3 | 3.070 |
| $\mathrm{Si}(7)-\mathrm{O}(10)-\mathrm{Si}(2)$ | 147.0 | 3.058 |
| $\mathrm{Si}(8)-\mathrm{O}(11)-\mathrm{Si}(1)$ | 145.6 | 3.064 |
| $\mathrm{Si}(1)-\mathrm{O}(12)-\mathrm{Si}(8)$ | 142.7 | 3.031 |


| $\mathrm{Si}(1)-\mathrm{O}(13)-\mathrm{Si}(3)$ | 151.2 | 3.095 |
| :---: | :---: | :---: |
| $\mathrm{Si}(6)-\mathrm{O}(14)-\mathrm{Si}(3)$ | 145.2 | 3.064 |
| $\mathrm{Si}(8)-\mathrm{O}(15)-\mathrm{Si}(2)$ | 153.7 | 3. 103 |
| $\mathrm{Si}(2)-\mathrm{O}(16)-\mathrm{Si}(4)$ | 155.9 | $3 \cdot 110$ |
| $\mathrm{Si}(5)-\mathrm{O}(17)-\mathrm{Si}(7)$ | 179.1 | $3 \cdot 162$ |
| $\mathrm{Si}(7)-\mathrm{O}(18)-\mathrm{Si}(9)$ | 155.3 | $3 \cdot 111$ |
| $\mathrm{Si}(5)-\mathrm{O}(19)-\mathrm{Si}(4)$ | 155.5 | $3 \cdot 117$ |
| $\mathrm{Si}(12)-\mathrm{O}(20)-\mathrm{Si}(11)$ | 148.1 | 3.081 |
| $\mathrm{Si}(12)-\mathrm{O}(21)-\mathrm{Si}(10)$ | 149.8 | 3.085 |
| $\mathrm{Si}(10)-\mathrm{O}(22)-\mathrm{Si}(1)$ | 155.4 | $3 \cdot 110$ |
| $\mathrm{Si}(6)-\mathrm{O}(23)-\mathrm{Si}(8)$ | 144.8 | 3.056 |
| $\mathrm{Si}(9)-\mathrm{O}(24)-\mathrm{Si}(11)$ | $150 \cdot 8$ | 3.093 |
| means | $150 \cdot 0$ | 3.079 |

## Angle Si-O-Si versus distance Si-O

In numerous crystal structures individual bond lengths $\mathrm{Si}-\mathrm{O}$ depend on the bond-strength variation $\Delta p(\mathrm{O})$ of the O atoms (Baur, 1970, 1976). In cases where $\Delta p(\mathrm{O})=0.0$ the bond-length variation obviously cannot depend on $\Lambda p(\mathrm{O})$; such instances are useful for studying other bonding effects (Baur, 1971). In tridymite there are 12 crystallographically independent four-connected $\mathrm{SiO}_{4}$ tetrahedra in which all O atoms are valence-balanced; that is their $\Delta p(\mathrm{O})$ is zero. These 12 tetrahedra can be added to the five such tetrahedra previously known in the literature (see Table 3) in order to test the dependence of individual $\mathrm{Si}-\mathrm{O}$ bond lengths on the negative secant of the bridging angle $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$, as discussed by Gibbs, Hamil, Louisnathan, Barteil \& Yow (1972). In addition this correlation was tested for a data set which included not only the tetrahedra from tridymite, low quartz, low cristobalite, coesite and zunyite, but also $\mathrm{Si}-\mathrm{O}$ bonds to charge-balanced O atoms in several disilicates, ring silicates and the remaining $\mathrm{Si}-\mathrm{O}$ bond in zunyite (Table 3). In either case the correlation coefficients are small (13 and 19\%)
and the slope of the $\mathrm{Si}-\mathrm{O}$ bond length with the negative secant (see Table 4) is much smaller than found by Gibbs et al. (1972). The null hypothesis (that the correlation coefficient and the slope equal zero) can be accepted at the 0.025 risk level because neither value of $|t|$ exceeds $t(77,0.05) \sim t(66,0.025) \simeq 1.99$ (one-tailed test). For the larger sample, $|t|$ barely reaches $t(77,0.05) \simeq 1 \cdot 67$. This is a small consolation since less than $4 \%$ of the variations in $\mathrm{Si}-\mathrm{O}$ are explained by the dependence on $\sec (\mathrm{Si}-\mathrm{O}-\mathrm{Si})$. The difference between the two results is partly due to the inclusion of the zunyite data (Louisnathan \& Gibbs, 1972), which were not present in the data set analyzed by Gibbs et al. Actually the correlation would be fair if the data for zunyite, $\mathrm{Er}_{2} \mathrm{Si}_{2} \mathrm{O}_{7}$ and $\mathrm{Yb}_{2} \mathrm{Si}_{2} \mathrm{O}_{7}$ were not included (Fig. 1). Considering the presently available empirical evidence, however, the $\mathrm{Si}-\mathrm{O}$ vs $\sec (\mathrm{Si}-\mathrm{O}-\mathrm{Si})$ correlation cannot be viewed as a general property of charge-balanced bridging O atoms in silicates.

O'Keeffe \& Hyde (1976) have recently pointed out that bridging angles between tetrahedral groups of Si and other first, second and third row elements in structures of cristobalite type depend mostly on the

Table 3. Individual distances $\mathrm{Si}-\mathrm{O}$ and $\mathrm{Si-Si}(\AA)$ and angles $\mathrm{Si}-\mathrm{O}-\mathrm{Si}\left({ }^{\circ}\right)$ in crystal structures in which all O atoms are charge-balanced $\lfloor\Delta p(\mathrm{O})=0.0 \mid$

The connectivity of the $\mathrm{SiO}_{4}$ tetrahedra, NC , is the number of bridging $(\mathrm{Si}-\mathrm{O}-\mathrm{Si}) \mathrm{O}$ atoms per tetrahedron. The estimated standard deviations of all $\mathrm{Si}-\mathrm{O}$ bonds are $0.010 \AA$ or smaller. All distances and angles have been recalculated from the positional parameters etc., in the original papers.

| Compound | $\mathrm{Si}-\mathrm{O}$ | $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ | $\mathrm{Si}-\mathrm{Si}$ | NC | Compound | $\mathrm{Si}-\mathrm{O}$ | $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ | $\mathrm{Si} \cdots \mathrm{Si}$ | NC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cristobalite ${ }^{1}$ | \{ 1.601 | 146.8 | 3.076 | 4 | Zunyite ${ }^{4}$ | $\{1.628$ | $180 \cdot 0$ | $3 \cdot 258$ | 4 |
|  | \{ 1.608 | $146 \cdot 8$ | 3.076 | 4 |  | \{1.630 | $180 \cdot 0$ | 3.258 | 1 |
| Low quartz ${ }^{2}$ | $\{1.603$ | 143.6 | 3.057 | 4 | $\mathrm{Yb}_{2} \mathrm{Si}_{2} \mathrm{O}_{7}{ }^{5}$ | 1.626 | 180.0 | 3.252 | 1 |
|  | 11.615 | $143 \cdot 6$ | 3.057 | 4 | $\mathrm{Er}_{2} \mathrm{Si}_{2} \mathrm{O}_{7}{ }^{5}$ | 1.632 | $180 \cdot 0$ | $3 \cdot 265$ | 1 |
| Coesite ${ }^{3}$ | ( 1.598 | 180.0 | 3.196 | 4 | $\mathrm{Sc}_{2} \mathrm{Si}_{2} \mathrm{O}_{7}{ }^{6}$ | 1.604 | $180 \cdot 0$ | $3 \cdot 208$ | 1 |
|  | 1.608 | 143.8 | 3.056 | 4 | Hemimorphite ${ }^{7}$ | 1.627 | 150.4 | 3.145 | 1 |
|  | 1.607 | 145.0 | 3.077 | 4 | Benitoite ${ }^{8}$ | $\left\{\begin{array}{l}1.630 \\ 1.648\end{array}\right.$ | 132.9 | 3.005 | 2 |
|  | 1.619 | 145.0 | 3.077 | 4 |  | 1.648 | 132.9 | 3.005 | 2 |
|  | 1.598 | $150 \cdot 5$ | 3.116 | 4 | Beryl ${ }^{9}$ | \{ 1.594 | 168.8 | $3 \cdot 174$ | 2 |
|  | 1.625 | $150 \cdot 5$ | $3 \cdot 116$ | 4 |  | 11.595 | 168.8 | $3 \cdot 174$ | 2 |
|  | 1.631 | $136 \cdot 1$ | 3.012 | 4 | Emerald ${ }^{9}$ | $\{1.585$ | 169.3 | $3 \cdot 176$ | 2 |
|  | ( 1.617 | $136 \cdot 1$ | 3.012 | 4 |  | 1 1.604 | 169.3 | $3 \cdot 176$ | 2 |

References: (1) Dollase (1965). (2) Zachariasen \& Plettinger (1965). (3) Araki \& Zoltai (1969). (4) Louisnathan \& Gibbs (1972). (5) Smolin \& Shepelev (1970). (6) Smolin, Shepelev \& Titov (1973). (7) McDonald \& Cruickshank (1967). (8) Fischer (1969). (9) Gibbs. Breck \& Meagher (1968).

Table 4. Regression equations for $\mathrm{Si}-\mathrm{O}$ on $-\mathrm{sec}(\mathrm{Si}-\mathrm{O}-\mathrm{Si})$ : source of data, sample size, intercept (a), slope (b), correlation coefficient $(r)$, percent variation explained (\%), $|t|$

| Data source | \| Tables 2 and 3 |  | Gibbs et al. (1972) |  |
| :---: | :---: | :---: | :---: | :---: |
|  | ( $\mathrm{NC}=4$ | $\mathrm{NC}=1,2$ and 4 | $\mathrm{NC}=4$ | $\mathrm{NC}=1,2$ and 4 |
| Sample size | 68 | 79 | 12 | 27 |
| $a$ | 1.578 (23) | 1.573 (19) | 1.539 | 1.545 |
| $b$ | 0.020 (19) | 0.027 (16) | 0.06 (2) | 0.06 (2) |
| $r$ | 0.126 | $0 \cdot 187$ | $0 \cdot 57$ | $0 \cdot 58$ |
| \% | 1.6 | 3.5 | 32 | 34 |
| $\|t\|$ | 1.03 | $1 \cdot 67$ | $2 \cdot 3$ | 3.6 |

non-bonded cation-cation contacts lbased on Glidewell's (1975) radii]. In order to test this hypothesis the shortest possible $\mathrm{Si}-\mathrm{Si}$ distance between neighboring tetrahedra must be determined. It occurs in benitoite, which is not surprising since, in the tight curvature of the $\mathrm{Si}_{3} \mathrm{O}_{9}$ rings, the Si atoms are forced together more closely than in most other silicate geometries. Therefore, if we assume (a) that Si atoms in silicates with charge-balanced O atoms cannot approach each other closer than $3.0 \AA$ and (b) that both $\mathrm{Si}-\mathrm{O}$ bonds in the $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ bridge are of the same length, then no data points should occur


Fig. 1. $\mathrm{Si}-\mathrm{O}$ bond lengths versus $-\sec (\mathrm{Si}-\mathrm{O}-\mathrm{Si})$ for 17 four-connected tetrahedra and $11 \mathrm{Si}-\mathrm{O}$ bonds of other bridging O atoms. The data are from Tables 2 and 3. The dashed line corresponds to the regression equation for the sample consisting of 79 values (see Table 4 ). For the explanation of the dotted lines see text.
below the dotted diagonal curve in Fig. 1. An upper limit for possible data points is given by the dotted horizontal line in Fig. 1 at $\mathrm{Si}-\mathrm{O}=1.64 \AA$, because $\mathrm{Si}-\mathrm{O}$ distances involving charge-balanced O atoms rarely exceed this value. The only data outside these limits are the benitoite points and one coesite point, which do not conform to assumption (b). The averages of the two $\mathrm{Si}-\mathrm{O}$ bridging bonds in both cases fall within the bounds. It is particularly impressive that the longest $\mathrm{Si}-\mathrm{O}$ distances of approximately $1.63 \AA$ occur over the whole range of bridging angles from 180 to $133^{\circ}$ with essentially constant values. It is equally interesting to see that straight $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ bonds can occur for $\mathrm{Si}-\mathrm{O}$ bonds ranging from 1.576 to $1.632 \AA$. This covers almost the whole range of $\mathrm{Si}-\mathrm{O}$ bond lengths, since only one $\mathrm{Si}-\mathrm{O}$ bond in the whole sample is outside these limits. The 79 data points are distributed over all the allowed space; however, they are not uniformly distributed. Therefore, regression calculations, especially when they ignore points in the upper left of the diagram, tend to give positive correlations between $\mathrm{Si}-\mathrm{O}$ and $-\mathrm{sec}(\mathrm{Si}-\mathrm{O}-\mathrm{Si})$. The diagonal dotted curve in Fig. 1 is calculated from the general expression relating one angle to the three sides of a triangle. Since this expression is, within the limits of interest, not far from being linear with the secant of the $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ angle, it is not surprising that Gibbs et al. (1972) found a linear relationship between $\mathrm{Si}-\mathrm{O}$ and the secant of $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$. Analysis of small samples which have no or few values with long $\mathrm{Si}-\mathrm{O}$ distances at straight or nearly straight $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ angles yields significant correlations between $\mathrm{Si}-\mathrm{O}$ and $-\sec (\mathrm{Si}-\mathrm{O}-\mathrm{Si})$ (Prewitt, Baldwin \& Gibbs, 1974). Such correlations disappear when larger, less biased samples are studied. For cases where the $\mathrm{Si}-\mathrm{Si}$ distance is close to the nonbonding contact length of $3.0 \AA$ the correlation between $\mathrm{Si}-\mathrm{O}$ distances and $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ angles does exist because of the dependence of the angle $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ on the $\mathrm{Si}-\mathrm{O}$ distances and not vice versa as required by $\pi$-bonding theory (Glidewell, 1975). Therefore it appears that the approach of O'Keeffe \& Hyde (1976) is useful to obtain a lower limit of possible $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ angles, even though it cannot give us an upper limit. For $\mathrm{Si}-\mathrm{O}$ bonds ranging in length from 1.576 to $1.632 \AA$ the upper limit of $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ is empirically found to be $180^{\circ}$. Angles larger than the lower limit may be imposed by packing constraints of the crystal structures (Baur, 1972).

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Note added in proof:—After this paper was submitted for publication a short communication by Glidewell (1977) was published. The non-bonded atomic radius of Si has been used there to rationalize the $\mathrm{Si}-\mathrm{O}-\mathrm{Si}$ angles in the silica polymorphs.

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